

# **Bimodal analysis of the directional wave spectra of some sea storms recorded offshore the Italian coasts**

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## **ABSTRACT**

This paper deals with the bimodality of the directional wave spectra, which can occur when swell and sea waves coming from different directions are superimposed.

In order to study the bimodality of the directional spreading function, a lot of directional wave spectra recorded by the Italian Wave Measurement Network at the stations of Crotone (offshore the Jonian Calabrian coast) and Mazara del Vallo (offshore the coasts of Sicily) were analyzed. These sites can be characterized, due to their geographical position, by the occurrence of bimodal sea states. The analysis consisted in a preliminary selection of the sea states, after which a comparison of the efficiency of some criteria to detect the bimodality of the sea states was made. The results show that only the site of Crotone exhibits some bimodal spectra, for which the different bimodality criteria have been applied and compared.

## 1 INTRODUCTION

Information on the directional characteristics of wind waves is of a great importance for a wide range of engineering and scientific purposes, e. g. calculation of sediment transport, evaluation of wave forces on offshore structures, etc.

Although for many applications involving wave directionality the mean wave direction and the directional width per frequency provide quite sufficient information, there are some applications (e.g. the comparison between the SAR and the buoy measurements) for which a detailed description of the whole directional spectrum is required.

The wave directional spectrum is generally expressed as:

$$S(f, \vartheta) = S(f) \cdot D(f, \vartheta) \quad (1)$$

in which  $D(f, \vartheta)$  is the directional spreading function and  $S(f)$  is the frequency spectrum, given by:

$$S(f) = \int_0^{2\pi} S(f, \vartheta) d\vartheta \quad (2)$$

The directional spreading function  $D(f, \theta)$  has the following properties:

$$\left. \begin{array}{l} D(f, \vartheta) \geq 0 \\ D(f, \vartheta + 2\pi) = D(f, \vartheta) \end{array} \right\} \text{for } -\infty < \vartheta < +\infty \quad (3)$$

and for any interval with length  $2\pi$ :

$$\int_{\alpha-\pi}^{\alpha+\pi} D(f, \vartheta) d\vartheta = 1 \quad \text{for } -\infty < \alpha < +\infty \quad (4)$$

The directional spreading function has been widely parametrized through on a priori-assumed shape. A frequently used two-parameter model is the one suggested by Longuet-Higgins et al.(1963):

$$D(f, \vartheta) = A(f) \cos^{2s} [(\vartheta - \vartheta_m) / 2] \quad (5)$$

in which:

- $\theta_m$  is the mean wave direction;
- $A(f)$  is a normalization constant;
- $s$  is the spreading index, which controls the directional width of  $D(f, \theta)$ .

The parameter  $s$  is maximum at the spectral peak, diminishes towards smaller and greater frequencies than the peak frequency.

Mitsuyasu (1975) showed that  $s_{max}$  increases with the decrease of the parameter

$$2\pi f_p U / g \quad (6)$$

which represents the growth stage of the wind waves. The Author introduced the following dependence for  $s_{max}$ :

$$s_{max} = 11.5(2\pi f_p \frac{U}{g})^{-2.5} \quad (7)$$

in which  $U$  is the wind speed.

Goda (1981) gave the following values for  $s_{max}$ :

$s_{max}=10$  for wind-waves;

$s_{max}=25$  for short- travelled swell;

$s_{max}=75$  for long- travelled swell;

Mitsuyasu(1975) gave also the following expression for  $D(f, \theta)$ :

$$D(f, \vartheta) = G(s) \cos[(\vartheta - \vartheta_m) / 2]^{2s} \quad (8)$$

in which:

$$G(s) = \frac{1}{\pi} 2^{2s-1} \frac{\Gamma^2(s+1)}{\Gamma(2s+1)} \quad (9)$$

Fig. 1 shows the shape of  $D(f, \theta)$  for different values of  $s$ .

Starting from the buoy measurements,  $D(f, \theta)$  can be obtained through a Fourier series truncated at the first two terms of the Fourier coefficients:

$$D(f, \vartheta) = \frac{1}{\pi} \left\{ \frac{1}{2} + \sum_{n=1}^2 [a_n(f) \cos(n\vartheta) + b_n(f) \sin(n\vartheta)] \right\} \quad (10)$$

The coefficients  $a_1, b_1, a_2, b_2$  are the four lowest Fourier coefficients of  $D(f, \theta)$ , which can be obtained from the variances of the buoy displacements by the following expressions:

$$a_1(f) = \frac{Q_{12}}{\sqrt{A_{11} \cdot A_{22} + A_{11} \cdot A_{33}}} \quad b_1(f) = \frac{Q_{23}}{\sqrt{A_{11} \cdot A_{22} + A_{11} \cdot A_{33}}} \quad (11)$$

$$a_2(f) = \frac{A_{22} - A_{33}}{A_{22} + A_{33}} \quad b_2(f) = \frac{2C_{23}}{A_{22} + A_{33}} \quad (12)$$

in which  $A_{11}$ ,  $A_{22}$ , and  $A_{33}$  are the variances of the vertical, S-N and W-E displacements,  $C_{23}$  is the covariance of the N-W displacements,  $Q_{12}$  and  $Q_{13}$  are the quadrature variances of the Z-N and Z-W displacements.

For most practical applications the following parameters obtained from  $D(f, \theta)$  can be used:

$$\text{Directional Spread} \quad S_p(f) = \sqrt{2(1 - m_1(f))} \quad (13)$$

$$\text{Skewness} \quad S_k(f) = \frac{n_2(f)}{\{0.5[1 - m_2(f)]\}^{3/2}} \quad (14)$$

$$\text{Kurtosis } K_u(f) = \frac{6 - 8m_1(f) + 2m_2(f)}{\{2[1 - m_1(f)]\}^2} \quad (15)$$

In which  $m_1, m_2, n_2$  are the circular moments given by

$$\begin{aligned} m_1(f) &= \sqrt{a_1^2(f) + b_1^2(f)} \\ m_2(f) &= a_2(f) \cos[2\vartheta_m(f)] + b_2(f) \sin[2\vartheta_m(f)] \\ n_2(f) &= -a_2(f) \sin[2\vartheta_m(f)] + b_2(f) \cos[2\vartheta_m(f)] \end{aligned} \quad (16)$$

Once  $a_1(f)$ ,  $b_1(f)$  and  $b_2(f)$  have been calculated on the basis of the buoy measurements,  $D(f, \theta)$  is obtained by eq.(10). An example of  $D(f, \theta)$  is given in fig.2.

A principal lobe and a secondary lobe are evident, which maxima are located for different values of  $\theta$ . If the areas associated with each lobe are comparable, the sea state is bimodal (usually a superposition of local sea waves and distant swell).

The routine software of the wave buoys doesn't give  $D(f, \theta)$  directly but it calculates the integral parameters  $\theta_m(f)$ ,  $S_k(f)$ ,  $K_u(f)$ ,  $S_p(f)$  obtained with the hypothesis of symmetrical and unimodal  $D(f, \theta)$ . So some criteria have been proposed to detect the bimodality of  $D(f, \theta)$  on the basis of the values of these and other integral parameters.

In this paper some of these criteria have been applied in order to test their efficiency to detect the bimodality of  $D(f, \theta)$ .

## 2 CRITERIA TO IDENTIFY THE BIMODALITY OF THE SEA STATES

One of the first parameters involving the directionality of the sea states was the long-crestedness introduced by Longuet-Higgins (1956):

$$\Gamma = \left[ \frac{1 - \sqrt{a_2^2 + b_2^2}}{1 + \sqrt{a_2^2 + b_2^2}} \right]^{1/2} \quad (17)$$

The long-crestedness varies between zero for unimodal waves and the unity for waves with directional homogeneity.

Goda et al. (1981) examined the principal direction and the long-crestedness in detail, and suggested that the long-crestedness isn't an index of the bimodality of the directional spectra. The Authors proposed an alternative parameter named the mean spreading angle  $\theta_k(f)$ :

$$\vartheta_k(f) = \tan^{-1} \left[ \frac{0.5b_1^2(1+a_2) - a_1b_1b_2 + 0.5a_1^2(1-a_2)}{a_1^2 + b_1^2} \right] \quad (18)$$

$\theta_k(f)$  varies from zero to 0.5 if the angles are expressed in radians.

Kobune et al.(1985) proposed a combination of the parameters  $I(f)$  and  $\theta_k(f)$  to detect the bimodality of the sea states.

Another bimodality criterion was introduced by Nwogu et al. (1987) on the basis of the values assumed by the parameter:

$$\beta(f) = \frac{\int_0^{2\pi} D(f, \vartheta) \cdot \cos[\vartheta - \vartheta_m] d\vartheta}{\int_0^{2\pi} D(f, \vartheta) \cdot \cos[2 \cdot (\vartheta - \vartheta_m)] d\vartheta} \quad (19)$$

The values of  $\beta(f)$  are higher than zero for unimodal sea states, lower than zero for bimodal ones.

The last criterion here examined was introduced by Kuik et al. (1988) who used a combination of the skewness  $S_k(f)$  and the kurtosis  $K_u(f)$ . High values of the skewness and low values of the kurtosis are associated with a bimodal  $D(f, \theta)$ , so the Authors defined an unimodal and symmetrical domain of the  $D(f, \theta)$  limited by the following values of  $K_u(f)$ :

$$\begin{aligned} K_u(f) &= 2 + S_k(f) \quad \text{for } |S_k(f)| \leq 4 \\ K_u(f) &= 6 \quad \text{for } |S_k(f)| > 4 \end{aligned} \quad (20)$$

The experimental points which exhibit higher values of the kurtosis are associated with a symmetrical and unimodal  $D(f, \theta)$ .

In this paper the efficiency of these criteria to detect the bimodality of the sea states has been compared on the basis of the analysis of the directional wave records.

### 3 DATA ANALYSIS AND RESULTS

The directional wave spectra of some sea storms recorded by the Italian Wave Measurement Network at the stations of Crotone and Mazara del Vallo during the period 1989-1997 were analyzed.

The wave measurement system is made with Directional Wavec buoys which geographical coordinates are listed in table 1.

STATION	DEPTH (m)	LAT.	LONG.
Crotone	Approx. 100	39°01',4 N	17°13',2 E
Mazara	Approx. 100	37°31',5 N	12°32',0 E

Table 1- Location of the wave measurement stations

A preliminary selection of the sea states with possible occurrence of bimodality was made on the basis of the values assumed by the long-crestedness  $\Gamma(f_p)$  and the mean spreading angle  $\theta_k(f_p)$ . Fig.3 shows the time history of these two parameters for the sea states recorded at Crotone station in the period 29/10/97-31/10/97.

The observed values of  $\Gamma$  are higher than the ones of  $\theta_k$  except for a few data, which are associated with a bimodal spectrum (two directional peaks) which can be noted from the contour plots in fig.4.

The selected sea states have been analyzed with the other two criteria for the frequencies bounded with  $f_p$  and  $2f_p$ , in which the main part of the spectral energy is concentrated.

Fig. 5 shows the results of the application of the criterion based on the values assumed by the skewness  $S_k(f)$  and the kurtosis  $K_u(f)$  for the cited frequencies. The observed values superimposed on the theoretical domain show that a lot of experimental points belong to the bimodal domain, which correspond to the the sea states with anomalous values of  $\Gamma(f_p)$  and  $\theta_k(f_p)$ .

Finally, fig. 6 shows the results of the application of the  $\beta$ -criterion for the same sea states and frequencies of fig.5. It can be noted that some experimental points belong to the bimodal domain, but their number is lower than the bimodal points of the other criterion. In other words a lot of experimental values of  $\beta$  belong to the unimodal domain ( $\beta(f) \geq 0$ ) in spite of their bimodal behaviour according to the criterion of fig. 5.

## 4 CONCLUSIONS

The preliminary analysis of some sea storms recorded offshore the Jonian Calabrian coasts in order to test some criteria designed to detect the bimodality of the sea states showed the following main results:

- the criterion based on the simultaneous values assumed by the long-crestedness  $\Gamma(f_p)$  and the mean spreading angle  $\vartheta_k(f_p)$  seems to be capable to detect the bimodality of the sea states;
- the criterion based on the simultaneous values assumed by the skewness  $S_k(f)$  and the kurtosis  $K_u(f)$  between the peak frequency and twice the peak frequency seems to be efficient, because a lot of experimental points belong to the bimodal domain of  $D(f, \vartheta)$ ;
- the criterion based on the values of the parameter  $\beta(f)$  seems to be less efficient, because a lower number of experimental points is located in the bimodal domain; so in some cases this criterion may fail in the detection of bimodal sea states.

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